Efficient Decentralized Power Control via a Compensation Mechanism

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Abstract—A “mechanism” is a set of procedures and rules intended to lead selfish entities to a desirable outcome, “on their own”. This work applies a relatively simple mechanism, available in the economics literature, to achieve an efficient decentralized allocation of power among data-transmitting terminals. The resulting operating point is “efficient”, because terminals end up “fairly” compensating each other for the interference each one causes. The same ideas can be fruitfully applied in more general networks, and outside the engineering context.

I. INTRODUCTION

It has long been recognized that decentralized control algorithms offer many advantages over their centralized counterparts. Factors include complexity, signaling overhead, and unavailability of local information to a central controller. Besides, certain modern communication and/or computing paradigms, such as ad-hoc wireless networks, and peer-to-peer computing, are inherently decentralized, which make central controllers highly impractical, if not outright impossible to implement.

Game theory can be a useful paradigm for decentralized control. For instance, power control in wireless data applications has been formulated as a “game”; i.e., a situation in which each of several selfish agents choose a “strategy” in order to maximize its own “payoff”, which depends on the chosen strategies by all players. The strategy is a power level, and the payoff is quality-of-service (QoS) (e.g., bits per Joule), which depends on the choices of all terminals, because the power chosen by a terminal becomes interference for others. The actual decisions may be made by “software agents”, which may be controlled and/or tuned or trained by an actual human operator, acting on his self-interests. Or these agents could be entirely programmed by the network administrator to its own advantage. Either case can be formulated as a game.

A key solution concept is a Nash equilibrium (NE); i.e., an allocation (a strategy per player) such that no player would be better off by unilaterally changing strategy (say transmission power). In the data terminals game, it is well understood that, when transmission power is limited, a NE always exists [1], [2]. However, the terminals settle on power levels that are “too high”. In fact, NE’s are generally “inefficient”.

The challenge is to get selfish entities to reach a desirable operating point “on their own”. An option is a “mechanism”; i.e., a set of procedures, penalties and rewards. In order to achieve an efficient decentralized allocation of power among mutually interfering terminals, this work applies a relatively simple mechanism introduced in [3]. This mechanism requires a “transferable good” (e.g., money, or some form of service credits) with which terminals can compensate each other for the loss in performance each causes.

Below, the physical system model is built, an underlying economic model is discussed, and the compensating mechanism is specified. Then, the equilibrium allocation is described for the special case in which one terminal interferes with the other but not vice-versa. Finally, the results are discussed, and directions for ongoing and future research mentioned.

II. PHYSICAL MODEL

In this simple model, the following quantities and/or concepts are of interest:

i) \( N \) is the number of terminals transmitting data simultaneously to a CDMA base station. For most of the development, \( N = 2 \). Extensions are discussed at the end.

ii) \( R_i \) bits per second is the source data rate of terminal \( i \).

iii) \( R_\text{C} \) chips per second is the chip rate (“bandwidth”) of the channel, common to all terminals.

iv) \( G_i = R_\text{C}/R_i \) is the processing gain of terminal \( i \).

vii) \( f_S(x_i) \) is the probability of correct reception of a data packet as a function of the signal-to-interference ratio (SIR) at the receiver. It is assumed that all that is known about \( f \) is that its graphs has an “S-shape”, as shown in fig. 1. This should accommodate most physical layer configurations of practical interest. The technical characterization of a “S-curve” is given in [4]. Below, \( f(x) := f_S(x) - f_S(0) \) replaces \( f_S(x) \) to avoid certain technical problems [1].

\[
x_i := G_i\alpha_i, \text{ with } \alpha_i \text{ the carrier-to-interference ratio (CIR) of the receiver tuned to transmitter } i, \text{ defined by:}
\]

\[
\alpha_i = \frac{P_i h_i}{\sum_{j \neq i} P_j h_j + \sigma^2} = \frac{Q_i}{\sum_{j \neq i} Q_j + \sigma^2} = \frac{Q_i}{I_i} (1)
\]

In this expression, \( P_i \) is the transmitted power of terminal \( i \), \( h_i \) is the path gain from terminal \( i \) to the base station, and \( \sigma^2 \) is the noise power in the base station receiver. \( Q_i = P_i h_i \) is the received power at the base station in the signal transmitted.
by terminal \(i\), and, for notational convenience, \(I_i\) denotes the total level of interference experienced by terminal \(i\).

Fig. 1. A typical “corrected” frame-success function. \(x^*\) always maximizes \(f(x)/x\).

III. DECENTRALIZED BITS/JOULE MAXIMIZATION: NO MECHANISM

A. Solution

A game in which terminals carrying multi-rate traffic choose transmission power in order to maximize bits/Joule is analyzed in [1]. The key solution concept is a Nash equilibrium; i.e., an allocation from which no terminal would be better off by unilaterally “deviating” (changing its power level). Reference [1] shows that, with limited transmission power, an equilibrium always exists; and even with unlimited power, an equilibrium may exist if certain sum of simple terms is less than 1. This reference provides closed-form equilibrium conditions and power levels, derived from first principles, and shows that all terminals want the same signal-to-interference ratio (SIR). But because of power limitations, some terminals cannot reach the necessary power level and must operate at their maximal power. At equilibrium, a number of terminals transmit at full power, and the others achieve the same optimal SIR. This SIR can be shown in several ways. Reference [2] shows that if all the terminals operating at the optimal SIR lowered their equilibrium power levels by a certain fraction, the payoff of each terminal increases. Thus, equilibrium power levels are “too high”. An alternative way of testing whether the equilibrium power levels are efficient is to verify whether this allocation satisfies the first-order optimizing conditions for a Pareto-optimal allocation. The first-order necessary conditions for Pareto-efficiency are the same as those of an allocation that maximizes a weighted sum of the payoffs of the terminals [5, p. 332]. It can be verified that the equilibrium allocation fails the necessary conditions for a maximizer of the weighted sum of the terminals payoffs, and therefore is not Pareto-efficient.

IV. A SIMPLE EFFICIENCY-INDUCING MECHANISM

In order to achieve an efficient decentralized allocation of power among mutually interfering terminals, there must exist one transferable good (say money) with which terminals can compensate each other. Only two terminals are considered.

A. Economic Model

The basic economic model is that of partial-equilibrium analysis and a quasi-linear utility function, as discussed, for instance in [5, Ch. 10]. Each terminal is assumed to have both an energy budget, \(E_i\), and a monetary budget, \(D_i\). The terminal’s payoff is \(\beta_iB_i + y_i\), where (i) \(\beta_i\) is the monetary value to the terminal of one information bit successfully transferred, (ii) \(B_i\) is the (average) number of information bits the terminal gets to successfully transfer by the time its energy runs out, and (iii) \(y_i\) is the amount of money the terminal has left after compensation, and penalties are computed.

Without penalties and rewards, the terminal keeps its complete monetary budget, \(D_i\), intact. Thus, the terminal’s payoff is \(\beta_iB_i + D_i\). But when a mechanism is introduced, the second term becomes \(D_i + y_i\), and a monetary budget, \(D_i\), plus any reward/compensation received, minus any penalty/compensation paid. This will be further clarified below.

B. The compensation mechanism

The mechanism is implemented in two stages: (i) announcement: the terminals announce the prices \(c_{12}, c_{21}, c_{31}, c_{12}^2\), where the superscript indicates which terminal sets the price, and the subscript \(ij\) denotes that money flows from \(i\) to \(j\). (ii) choice: each terminal chooses its power level to maximize its payoff, given the announced prices. If the compensation offered by terminal \(i\) does not match what terminal \(j\) wants, terminal \(i\) must pay a penalty \((c_{ij} - c_{ij}^2)^2\) to a third party. Thus, once all choices have been made, the payoff to terminal \(1\) is:

\[
\frac{\beta_1}{2}B_1(P_1; P_2) + \frac{D_1}{2} + c_{21}^2P_2 - c_{12}^2P_1 - \left(\frac{c_{12} - c_{12}^2}{2}\right)
\]

$/$bits bits budget from 2 to 2 Penalty

C. Describing the equilibrium for the asymmetric case

Reference [3] shows that the allocation arising from this game is efficient. Nevertheless, it is interesting to describe the powers and prices arising at equilibrium. This is done below for the special case in which terminal 1 interferes with terminal 2 but not vice-versa (successive interference
cancellation (SIC) decoding). In this case, \(c_2\) denotes the unit compensation terminal 2 (“injured” terminal) demands, and \(c_1\) the compensation offered by terminal 1 (interferer). Since the injured terminal makes no payments, it is convenient to set its monetary budget \(D_2 = 0\).

1) Second-stage payoffs: After all choices have been made, the payoffs or the asymmetric game are:

\[
\begin{align*}
\beta_1 & \cdot B_1(P_1; I_1) + \frac{D_1}{b_2} - c_2 P_2 - (c_1 - c_2)^2 \\
\beta_2 & \cdot B_2(P_2; I_2) + c_1 P_1 - (c_2 - c_1)^2
\end{align*}
\]

For a given level of interference, \(I_i\)

\[
B_i(P_i, I_i) = E_i \frac{L}{M} R_i f(G_i h_i P_i / I_i) = R_i E_i \frac{L}{M} h_i f(x_i) x_i
\]

with \(x_i\) the signal to interference ratio (SIR) at the receiver \((L/M)\) is the ratio of information bits to total bits in a packet \([1]\).

2) Characterizing the equilibrium:

\[u(x) = u(x) - cx\]

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a) General approach: This is a 2 stage game. To solve it, one first looks at the second stage (choosing the power levels), as if the first-stage choices had been pre-determined. This gives power levels that are a function of \(c_1\) and \(c_2\). With this information, the first-stage of the game can be solved. But notice from eq. (3), that the choice of \(c_1\) only impacts the interferer’s payoff through the penalty term, \((c_2 - c_1)^2\). \(c_1\) does influence the power chosen by terminal 2, but this power has no effect on the interferer’s payoff because, by assumption, terminal 1’s only impairment is random noise. Thus, at equilibrium \(c_1 = c_2\) because it is always optimal for the interferer to avoid the penalty. Hence, in characterizing the equilibrium allocation, one can focus on the case \(c_1 = c_2 = c\).

b) Interferer’s power choice: By assumption, terminal 1 interferes terminal 2 but not vice-versa. That is, \(I_1 = \sigma^2\), while \(I_2 = P_2 + \sigma^2\).

Terminal 1 will choose \(P_1\) so that \(x_1\) maximizes

\[
R_i \frac{L}{M} E_i h_1 f(x_1) x_1 - c \frac{\sigma^2}{G_1 h_1} x_1 = \hat{\beta}_1 u(x_1) - \hat{c}_1 x_1
\]

where for notational convenience

\[
u(x_1) := f(x_1) \quad \hat{\beta}_1 := \frac{L R_i E_i h_1}{\sigma^2} \quad \hat{c}_1 := \frac{c \sigma^2}{G_1 h_1}
\]

Equation (6) has the general form \(u(x) - cx\).

c) Maximizing \(u(x) - cx\): Reference [4] shows that for \(f\) an S-curve, \(f(x)/x\) is “single peaked”, as shown in Fig. 2. Thus, the maximization of \(u(x) - cx\), where all that is known about \(u\) is that it is “single peaked”, needs to be understood.

As Fig. 2 clearly shows, if \(c\) exceeds certain critical value, \(c_L\), the line \(cx\) lies entirely over the curve \(u(x)\) except at the origin. Thus, \(u(x) - cx < 0\) for any positive \(x\), which implies that it is optimal for this terminal to set \(x = 0\). At the other extreme, if \(c \approx 0\), the maximum occurs at \(x^*\), which is shown in fig. 1 at the tangency point between \(f\) and a line from the origin[4]. For \(0 < c \leq c_L\), there is an interval \((a, b)\) on which \(u(x) > cx\) (when \(c = c_L\), a and b “merge” into \(x_L\)).

The function \(u(x) - cx\) is continuous; therefore it must have a maximum over the closed and bounded interval \([a, b]\). The maximum occurs at the point \(x^*\) where \(u'(x) = c\) (that is, a point at which a tangent to the curve is parallel to the line).

With power limitations, the terminal may not be able to exceed a certain maximal SIR \(\bar{x}\). In this case, if \(a < \bar{x} \leq x^*\) it is optimal for this terminal to operate at \(\bar{x}\). However, if \(\bar{x} < a\) the optimal choice for this terminal is 0, since \(u(x) - cx < 0\) for \(0 < x < a\). If \(\bar{x} = a\) the terminal is indifferent between choosing 0 or a. In the interest of simplicity, it is assumed that when operating and not operating give an identical utility, the terminal will choose to operate.

In conclusion, the problem of maximizing \(u(x) - cx\) is well defined, and has a solution. Depending upon the value of \(c\) and \(\bar{x}\), the maximizer could be 0, \(\bar{x}\), or \(x^*\). Thus, for fixed \(\bar{x}\), the function \(x(c)\) giving the maximizer of \(u(x) - cx\) is well-defined.

![Fig. 2](image-url)
shown in fig. 1 at the tangency point between \( f \) and a line from the origin[4].

e) Injured terminal’s price choice: Through the development in sections IV-C.2.b through IV-C.2.d, the second stage (“choice”) of the asymmetric compensation game has been characterized. The same must be done for the stage of the game in which terminals announce their prices. As remarked in section IV-C.2.a, at equilibrium the interferer’s compensation will match that demanded by the injured terminal. Thus, all that remains is to specify the price that terminal 2 will demand. This is done with the understanding that for any chosen compensation \( c \), the interferer will choose its power so that its received SIR is \( x_1(c) \) (section IV-C.2.c), and the injured terminal will choose to operate at the SIR \( x^* \) (section IV-C.2.d).

The injured terminal will choose \( c \) to maximize its overall utility (taking into account what will happen in the next stage of the game). That is, it will maximize

\[
v(c) := \frac{A}{h_1P_1(c) + \sigma^2} + cP_1(c) \tag{8}
\]

with

\[
A := R_e \frac{L}{M} h_2E_2\beta_2 \frac{f(x^*)}{x^*}
\]

\( v(c) \) is a single variable function which can be readily maximized, whether analytically or numerically. \( P_1(c) \) chooses the optimally chosen power of the interferer for any given \( c \), which follows from the analysis in section IV-C.2.c. Disregarding power constraints, the function \( x_1(c) \) (optimal SIR or the interferer) can be assumed to vary smoothly with \( c \) (i.e., to be continuously differentiable), over the interval \([0, c_L]\) (see fig. 2), and the same can be said about \( P_1(c) \). Over this range, \( v(c) \) is then a composite function of continuous functions, which is therefore continuous, and must have a maximum over a closed and bounded set. Therefore, \( c^* := \arg \max v(c) \) is well defined. Furthermore, with \( P_1(c) \) differentiable over \([0, c_L]\), the derivative of \( v(c) \) is well defined, and can be obtained and set to zero.

\( v(c^*) \) is the best the injured terminal can do, with a price low enough to induce the interferer to pay and operate. If \( c > c_L \) (say \( c = c_L + \epsilon \)), the interferer will choose \( not \) to operate, deriving a total utility of \( D_1 \), its original monetary budget. In this case, the interfered terminal will have the channel to itself, will receive no compensating money, and will get the the bits/Joule performance of a random noise channel (say \( B_0 \) total bits, given its energy budget). Thus, if terminal 2 sets \( c = c_L + \epsilon \), its utility will be \( \beta_2B_0 \). This must be compared against \( v(c^*) \), for a final choice of the price \( c \) (either \( c^* \) or \( c_L + \epsilon \)).

V. DISCUSSION

A compensation mechanism has been applied to the achieve an efficient allocation of power among two mutually-interfering, data-transmitting terminals. The mechanism is efficient because it induces the terminals to “fairly” compensate each other, by way of money or some transferable good. With 2 mutually interfering terminals, each terminal must quote two prices: one to be paid to the other as compensation; the second to be charged as compensation. But each terminal faces a penalty if its offered price differs from what the other wants as compensation.

The development provides further insights into the equilibrium allocation, for the special case in which terminal 1 interferes with terminal 2, but \( not \) vice-versa (SIC decoding). The interfered terminal will operate at the optimal SIR, \( x^* \) (fig. 1), which is the same value a terminal would choose with random noise as the only impairment. The interferer will either stay out of the channel, or pay the exact compensation price \( c \) demanded by terminal 2, and operate at the SIR \( x_1(c) \) illustrated in fig. 2. \( x_1(c) \) is always less than \( x^* \). The optimal \( c \) maximizes \( v(c) \), a relatively simple function (eq. (8)) which has a continuous derivative for values of \( c \) that are low enough to entice the interferer into operating.

A complete characterization and understanding of the power and money allocations arising from this mechanism necessitates additional analytical and numerical work. Likewise, its impact on several issues involving communication networks is being explored.

For instance, a mechanism such as this should induce a more judicious use of the network by terminals that are in “bad locations”. Likewise, if the mechanism is applied to many mutually-interfering terminals, the exchange of pricing signals between terminals becomes an issue. However, the fact that terminals only care about the total interference helps, because a terminal’s charge per unit of interference should be independent of the source of the interference. But each terminal may, in principle, quote a different value. The rate of convergence toward the equilibrium prices and power levels is also a concern. But it can be shown that a simple updating algorithm exists that leads to the equilibrium, even when terminals don’t know “everything” about each other. In an ad-hoc network, the main challenge may be to set up a practical accounting system to track down the compensating payments among terminals. These an other interesting issues will be addressed in future reports of this work.

REFERENCES