PREDICTIVE CONTROL OF HYBRID SYSTEMS UNDER A MULTI-MLD FORMALISM

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Abstract: The Mixed Logical Dynamical (MLD) formalism has proved to be an efficient modelling framework for hybrid systems described by dynamics, logic and constraints. Furthermore, it allows formulating and solving practical problems such as control, using for example predictive strategies. However, its main drawback remains the computation load due to the complexity of the MIQPs to be solved.

To overcome this problem, this paper presents a multi-MLD model, considering split state space regions inside which restricted simpler size MLD models take into account only variables variations that may occur. This approach enables to considerably reduce the computation time, to become more convenient for real time implementation even with small sampling time. This strategy is applied in simulation to the control of a three tanks benchmark. Copyright © 2003 IFAC

Keywords: Hybrid systems, Mixed logical dynamical systems, Model predictive control, Mixed integer quadratic programming, State space partition, Switched dynamics.

1. INTRODUCTION

Many control problems involve hybrid systems, including both continuous and discrete variables, discrete variables coming from parts described by logic such as for example on/off switches or valves. Various approaches have been proposed for modelling hybrid systems (Branicky et al., 1998), like Automata model, Petri nets model, Linear Complementary (LC) model. It was shown that moving logical relations into linear constraints on integer variables provides a global modelling framework called Mixed Logical Dynamical (MLD) formalism (Bemporad and Morari, 1999). It allows describing a large number of important classes of hybrid systems, such as piecewise linear systems, systems with mixed continuous/discrete inputs and states …

This formalism can also formulate and solve practical problems such as state estimation or control, and predictive strategies in that sense provide efficient tools, which enable MLD systems to track a desired reference trajectory.

The main drawback of this MLD formalism remains the computational burden related to the complexity of the derived MIQPs problems which depend on the complexity of the MLD model and the prediction horizon N. Indeed MIQPs problems are classified as NP-complete, so that in the worst case, the optimisation time grows exponentially with the problem size, even if branch and bounds methods may reduce this solution time (Fletcher and Leyffer, 1995). In order to reduce the computational complexity alternative approaches have been developed such as the one presented in (Stursberg and Engell 2002).

This paper examines a modified formulation of the MLD model, which efficiently decreases the computation time. The global state space domain is divided into separate regions inside which only feasible variables variations are considered. This leads for each region to the design of smaller size MLD model and MIQPs problems of restricted complexity.

The paper is organized as follows. Section 2 presents a description of the MLD systems. General consideration about model predictive control (MPC) and its application to MLD systems are developed in section 3. Section 4 examines the multi-MLD model. Finally, section 5 presents the application of this strategy to the water level control of a three tanks benchmark.
2. MLD MODEL

The Mixed Logical Dynamical (MLD) model permits the description of various classes of hybrid systems, like linear hybrid systems, constrained linear systems, sequential logical systems (finite state machines, automata), some classes of discrete event systems, and non-linear dynamic systems, where nonlinearities can be expressed through logical combination. The MLD model describes the systems by linear dynamic equations subject to linear inequalities involving both real and integer variables, under the following form (see Bemporad and Morari, 1999, for more details):

\[
\begin{align*}
\dot{x}(t+1) &= A x(t) + B_1 u(t) + B_2 \delta(t) + B_3 z(t) \\
y(t) &= C x(t) + D_1 u(t) + D_2 \delta(t) + D_3 z(t)
\end{align*}
\]

where:

\[
x = \begin{bmatrix} x_1 \\ \vdots \\ x_m \end{bmatrix} \in \mathbb{R}^{m_x} \times \{0,1\}^{m_l}, \quad u = \begin{bmatrix} u_1 \\ \vdots \\ u_m \end{bmatrix} \in \mathbb{R}^{m_u} \times \{0,1\}^{m_l},
\]

\[
y = \begin{bmatrix} y_1 \\ \vdots \\ y_n \end{bmatrix} \in \mathbb{R}^{n_y} \times \{0,1\}^{n_l}, \quad \delta = \begin{bmatrix} \delta_1 \\ \vdots \\ \delta_n \end{bmatrix} \in \{0,1\}^{n_l}, \quad z \in \mathbb{R}^n
\]

are respectively the vectors of continuous and binary states of the system, of continuous and binary (on/off) control inputs, of output signals, of auxiliary binary and continuous variables. The auxiliary variables are introduced when translating propositional logic into linear inequalities as described in Figure 1.

![Fig. 1. MLD model structure.](image)

The continuous input \( u_e \) is supposed to be bounded:

\[
U = \{ u_e \in \mathbb{R}^m | L u_e \leq p, L \in \mathbb{R}^{q \times m}, p \in \mathbb{R}^q \} 
\]

A MLD model Eq. 1 thus represents logical relations and interaction between continuous and logical variables by mixed integer linear inequalities. \( \{A, B_1, B_2, C, D_1, D_2, E \} \) matrices in Eq. 1 are obtained through the specification language HYSDEL (Hybrid System Description Language) as explained in (Bemporad and Mignone, 2000b).

3. MODEL PREDICTIVE CONTROL

Model predictive control (MPC) has proved to efficiently control a wide range of applications in industry. It is capable to control a great variety of processes, including systems with long delay times, non-minimum phase systems, unstable systems, multivariable systems, and constrained systems (Camacho and Bordons, 1999).

3.1 General consideration.

The main idea of predictive control is to use a model of the plant to predict future outputs of the system. Based on this prediction, at each sampling period, a sequence of future control values is elaborated through an on-line optimisation process, which maximizes the tracking performance while satisfying constraints. Only the first value of this optimal sequence is applied to the plant, the whole procedure is repeated again at the next sampling period according to the ‘receding’ horizon strategy (Boucher and Dumur, 1996).

The cost function to be minimized is generally a weighted sum of square predicted errors and square future control values, e.g. in Generalized Predictive Control (GPC) (Clarke, Mohtadi and Tuffs, 1987). For a certain class of hybrid systems an adaptive GPC has also been proposed (Buisson and Bergomi, 1994).

3.2 Model Predictive Control for MLD systems.

For an MLD system of the form Eq. 1, the following model predictive control problem is considered. Let \( t \) be the current time, \( x(t) \) the current state, \( (x_e, u_e) \) an equilibrium pair or a reference trajectory value, \( x_0 \) the initial state, \( T \) the final time, find \( u_0^{T-1} = (u(0) \cdots u(T-1)) \) the control sequence which moves the state from \( x_0 \) to \( x_e \) and minimizes the cost function:

\[
\min_{u_{0}^{T-1}} J(u_{0}^{N-1}, x_{0}) = \sum_{k=0}^{N-1} \left[ u(k) - u_0 \right]_e^2 + \\
+ \left[ \delta(k) - \delta_0 \right]_e^2 + \left[ \left| y(k) - y_0 \right|_e \right]_e^2
\]

subject to Eq. 1, and \( u(t) \) constant for \( k \geq N_u \), where \( N \) is the prediction horizon, \( N_u \) the control horizon, \( \delta_e, z_e \) are the auxiliary variables of the equilibrium point or the reference trajectory value, calculated by solving a MILP problem for the inequality equation. \( x(k/T) = x(t+k), x(t), u_0^{k-1} \) (in a similar way for the other input and output variables), \( Q_i = Q_i > 0 \), for \( i = 1,4 \), and \( Q_2 = Q_2 \geq 0 \), \( i = 2,3,5 \).

It must be noticed that this form Eq. 3 is slightly different from the one proposed by (Bemporad and Morari, 1999), as the final equality condition \( x(t+T) = x_e \) has been cancelled to be sure that the optimiser can find a solution whatever the prediction horizon may be (even short). Moreover, \( N_u \) has been added to limit the future control values in order to decrease the number of optimisation variables.

3.3 Mixed integer quadratic programming (MIQP).
The optimisation procedure of Eq. 3 leads to MIQP problems under the form:

$$\min_{\chi{\in}} \chi^T H \chi + \chi^T \varphi$$

subject to $c^T \chi \leq b$ (4)

where the optimisation vector is:

$$\chi = [ u(k), \cdots, u(k+N-1), \delta(k), \cdots, \delta(k+N-1), z(k), \cdots, z(k+N-1)]^T$$

and the number of binary optimisation variables is $L = N (m_1 + r_j)$. In the worst case the optimisation time increases exponentially with the number of binary optimisation variables (Raman and Grossmann, 1991). From this point of view, a unique MLD model describing the complete behavior of the system overall the state space and including all the variables may lead to a large size model with a huge number of binary variables which causes a computational problem

4. MULTI MLD MODELS

The contribution of this paper, in order to simplify the original problem, consists in partitioning the continuous state space in domains where a subset of the original problem, consists in partitioning the continuous state space in domains where a subset of the variables may lead to a large size model with a huge number of binary variables which causes a computational problem.

4.1 State space partition.

This partitioning can be formally determined as follows. Let:

- $\Delta = \{ \delta_1, \ldots, \delta_n \}$ be the set of all the auxiliary variables,
- $\Delta_a = \{ \delta_{a,j_1}, \ldots, \delta_{a,j_\alpha} \}$ a subset of $\Delta$ containing $\alpha$ distinct elements, $j = 1 \cdots C^\alpha$, where $\delta_{a,j_1} = 1$ if and only if $F_{\alpha,j_1} x_c \geq d_{\alpha,j_1}$ (e.g. determined by physical consideration).
- $V^I \Delta_\alpha$ one of the $2^\alpha$ possible combinations of values of the elements of $\Delta_\alpha$; $V^I (\ell)$ will denote the value of $\delta_{a,j_1} \Delta_a$.

From this, $R \left( \Delta^L_{\alpha}, V^I \Delta_\alpha, N \right)$ is the domain of the continuous state space defined by:

$$R \left( \Delta^L_{\alpha}, V^I \Delta_\alpha, N \right) = \begin{cases} x_c (k) \in \mathbb{R}^N, & \forall u(k+1) \forall i = 1 \cdots N, \\ x_c (k+i) \in R \left( \Delta^L_{\alpha}, V^I \Delta_\alpha, 0 \right) \end{cases}$$

with:

$$x_c \in \mathbb{R}^N \quad \text{if} \quad V^I (i) = 1$$

$$R \left( \Delta^L_{\alpha}, V^I \Delta_\alpha, 0 \right) = \begin{cases} x_c \in \mathbb{R}^N, & \text{then} \quad F_{\alpha,j_1} x_c \geq d_{\alpha,j_1}, \\ x_c \in \mathbb{R}^N, & \text{else} \quad F_{\alpha,j_1} x_c \geq d_{\alpha,j_1}, i = 1 \cdots \alpha \end{cases}$$

which implies that a trajectory starting in this domain will induce constant $V^I \Delta_\alpha$ values for at least $N$ steps. This domain can be recursively defined for $K \leq N$:

$$R \left( \Delta^L_{\alpha}, V^I \Delta_\alpha, K \right) = \begin{cases} x_c \in \mathbb{R}^N, & \forall u(k) \forall i = 1 \cdots N, \\ x_c \in \mathbb{R}^N, & \forall u(k+1) \forall i = 1 \cdots N, \\ x_c \in \mathbb{R}^N, & \forall u(k+i) \forall i = 1 \cdots N, \\ x_c \in \mathbb{R}^N, & \text{then} \quad F_{\alpha,j_1} x_c \geq d_{\alpha,j_1}, \\ x_c \in \mathbb{R}^N, & \text{else} \quad F_{\alpha,j_1} x_c \geq d_{\alpha,j_1}, i = 1 \cdots \alpha \end{cases}$$

The exact computing of those domains requires reachability tools for MLDs and is the object of current studies (Torrisi, 2003). In practical cases, sub-approximations of those domains can be heuristically determined.

Fig. 2. State space partition.

Figure 2(a) presents a general situation where $K > K^*$, for two different subsets $\Delta'$ and $\Delta''$ with $R_{\Delta',K} \subset R_{\Delta'',K}$ for any $\Delta$. Figure 2(b) presents a case where $\Delta'' \subset \Delta'$ and where the common boolean auxiliary variables have the same value, which implies $R_{\Delta'',J} \subset R_{\Delta',J}$. Note that notations in this figure have been simplified for clarity reasons.

4.2 Multi MLD development.

For each previous region $R \left( \Delta^L_{\alpha}, V^I \Delta_\alpha, K \right)$, as some binary auxiliary variables are known, a simpler MLD model can be developed. It should be noticed that this model which is only valid for $K$ iterations, does not depend on $K$. Hence, the dynamics are expressed as:

$$\begin{align*}
\dot{x}(t) &= A_{\Delta} x(t) + B_{\Delta} u(t) + B_{\Delta} \delta(t) + B_{\Delta} z(t) \\
y(t) &= C_{\Delta} x(t) + D_{\Delta} u(t) + D_{\Delta} \delta(t) + D_{\Delta} z(t) \\
E_{\Delta} \delta(t) + E_{\Delta} z(t) &\leq E_{\Delta} u(t) + E_{\Delta} x(t) + E_{\Delta}
\end{align*}$$

(5)
Where $\delta_i$ is composed with the binary auxiliary variables which do not belong to $\mathcal{A}^*_k$ and:

$$u_i \in \mathbb{R}^{m_i} \times \{0,1\}^{m_i}, m_i c \leq m_c, m_i l \leq m_l,$$

$$z_i \in \mathbb{R}^{r_i}, r_i c \leq r_c$$

Two main strategies can be used to build the MIQP that must be solved at time $k$.

- Look for all the $R\left(\mathcal{A}_k^i, V_k^i, N\right)$ containing $x_i(k)$, select the simplest model (which is the model with the smallest number of binary variables) corresponding to one of these domains and use it to build the MIQP problem, as in 3.3 with this single model.

- Look for all the $R\left(\mathcal{A}_k^j, V_k^j, i\right)$ containing $x_i(k)$, select the simplest model corresponding to one of these domains for the prediction at time $k+i$, and use it to build the MIQP problem, as in 3.3 with those multiple models.

For the example of fig 2 (a) suppose that $x_i(k) \in R\mathcal{A}_k^i \cap R\mathcal{A}_k^j \cap R\mathcal{A}_k^k$ and that the model corresponding to $\mathcal{A}^j$ is simpler than that of $\mathcal{A}^i$ and that $K = N$. The first strategy will lead to use only the model corresponding to $\mathcal{A}^j$ while the second strategy allows to use the model corresponding to $\mathcal{A}^i$ for prediction over $k, \ldots, k+K'$ and the model corresponding to $\mathcal{A}^k$ for prediction over $k+K'+1, \ldots, k+N$.

5. APPLICATION

The proposed control strategy is applied on the three tanks benchmark used by (Bemporad et al., 1999).

5.1 Description of the benchmark.

The simplified physical description of the three tanks system proposed by COSY as a standard benchmark for control and fault detection problems is presented in Figure 3 (see Dolanc et al., 1997, for more details).

![COSY three tank benchmark system](Fig. 3. COSY three tank benchmark system)

The system consists of three tanks, filled with water by two independent pumps acting on tanks 1 and 2. These two pumps are continuously manipulated from 0 up to a maximum flow $Q_1$ and $Q_2$ respectively. Four switching valves $V_1$, $V_2$, $V_{13}$ and $V_{23}$ control the flow between the tanks, those valves are assumed to be either completely opened or closed ($V_i = 0$ or 1 respectively). The $V_{N3}$ manual valve controls the nominal outflow of the middle tank. It will be assumed in further simulations that the $V_{L1}$ and $V_{L2}$ valves are always closed and $V_{N3}$ is open. The liquid levels to be controlled are denoted $h_1$, $h_2$ and $h_3$ for each tank respectively.

The conservation of mass in the tanks provides the following differential equations:

$$\begin{align*}
\dot{h}_1 &= \frac{1}{A}(Q_1 - Q_{13V1} - Q_{13V13}) \\
\dot{h}_2 &= \frac{1}{A}(Q_2 - Q_{23V2} - Q_{23V23}) \\
\dot{h}_3 &= \frac{1}{A}(Q_{13V1} + Q_{13V13} + Q_{23V2} + Q_{23V23} - Q_N) 
\end{align*}$$

(6)

where the $Q$'s denote the flows and $A$ is the cross-sectional area of each of the tanks. The Toricelli’s law provides the expressions of the flows through the valves, which are given under a linearized form by the relations:

$$\begin{align*}
Q_{13V1} &= k_{i3} V_{i3} (h_1 - h_3) \\
Q_{13V13} &= k_{i} V_{i} (\max(h_v, h_1) - \max(h_v, h_3)) \\
Q_{N3} &= k_{N3} V_{N3} h_3 
\end{align*}$$

(7)

where:

$$k_{i3} = a_z S_{i3} \sqrt{\frac{2g}{h_{\text{max}}}}, \quad k_{N3} = a_z S_{N3} \sqrt{\frac{2g}{h_{\text{max}}}}$$

From these expressions, a MLD model is derived as developed in (Bemporad et al., 1999), introducing the following variables:

$$\begin{align*}
x &= [h_1 \quad h_2 \quad h_3]' \\
u &= [Q_1 \quad Q_2 \quad V_1 \quad V_{13} \quad V_{23}]' \\
\delta &= [\delta_01 \quad \delta_02 \quad \delta_03]' \\
z &= [z_01 \quad z_02 \quad z_{03} \quad z_1 \quad z_2 \quad z_{13} \quad z_{23}]'
\end{align*}$$

(8)

where:

$$\begin{align*}
\delta_01(t) &= 1 \leftrightarrow [h_i(t) \geq h_i] \quad i = 1,2,3 \\
z_{0i}(t) &= \delta_01(t) (h_i(t) - h_i) \\
z_i(t) &= V_i (z_{0i}(t) - z_{03}(t)) \quad i = 1,2 \\
z_{i3}(t) &= V_{i3} (h_i(t) - h_3) \quad i = 1,2 
\end{align*}$$

(9)

5.2 Application of the multi MLD model

Let consider now the following specification: starting from zero levels (the three tanks being completely empty), the objective of the control strategy is to reach the liquid levels $h_1 = 0.5 \text{ m}$, $h_2 = 0.5 \text{ m}$ and $h_3 = 0.1 \text{ m}$. A comprehensive study of the dynamic
behaviour of the three tanks, starting from zero levels to the desired ones, enables to divide the state space into three main regions $R = [R_1, R_2, R_3]$, each one with its adequate simple MLD model.

The state space is characterized by the domain bounded by the level constraints:

$$X = R_1 \cup R_2 \cup R_3 = [0, 0.62] \times [0, 0.62] \times [0, 0.3]$$

(10)

With the three tank levels as state variables, the first region of the state space is characterized by:

$$R_1 = [0, 0.2] \times [0, 0.2] \times [0, 0.3]$$

(11)

In this region the auxiliary binary variables $\delta$ is completely determined ($\delta = [000^T]$), and it clearly appears that the two valves $V_1$ and $V_2$ of the input vector are not in progress, as the liquid level in this region is always less than the valves level. Consequently, the continuous auxiliary variables $\{q_0\} = i_{1,2,3}$ and $\{v_i\} = i_{1,2}$ corresponding to the flows that pass through the upper pipes are useless. It results from this a very simple $M_1$ model with the variables:

$$x = [ h_1 \ h_2 \ h_3 ]', \quad u_1 = [ \Omega_1 \ \Omega_2 \ V_{13} \ V_{23} ]', \quad \delta_1 = [ 1 ], \quad z_1 = [z_{13} \ z_{23} ]'$$

(12)

From the control viewpoint, this region can be split into two sub-regions as well, which enables to select adequate output and control prediction horizons:

$$R_{1,5} = [0, 0.08] \times [0, 0.08] \times [0, 0.3]$$

with $N = 5, N_u = 3$

$$R_{1,3} = R_1 - R_{1,5} \text{ with } N = 3, N_u = 3$$

The second region is characterized by:

$$R_2 = [0.38, 0.62] \times [0.38, 0.62] \times [0, 0.3]$$

(14)

In this second region $\delta = [110]'$, thus the $M_2$ model is related to the variables:

$$x = [ h_1 \ h_2 \ h_3 ]', \quad u_2 = [ \Omega_1 \ \Omega_2 \ V_1 \ V_2 \ V_{13} \ V_{23} ]'$$

(15)

$$\delta_2 = [ 1 ], \quad z_2 = [z_1 \ z_2 \ z_{13} \ z_{23} ]'$$

This region can also be divided into two sub-regions:

$$R_{2,5} = [0.43, 0.62] \times [0.43, 0.62] \times [0, 0.3]$$

with $N = 5, N_u = 2$

$$R_{2,3} = R_2 - R_{2,5} \text{ with } N = 3, N_u = 3$$

The third region is characterized by:

$$R_3 = X - R_1 - R_2 \text{ with } N = 2, N_u = 2$$

(17)

Switching can now occurs as the levels in the first and second tanks may pass the $h_3$ value. The third MLD model $M_3$ requests the following variables:

$$x = [ h_1 \ h_2 \ h_3 ]', \quad u_3 = [ \Omega_1 \ \Omega_2 \ V_1 \ V_2 \ V_{13} \ V_{23} ]'$$

$$\delta_3 = [\delta_{01} \ \delta_{02} ]', \quad z_3 = [z_1 \ z_2 \ z_{13} \ z_{23} ]'$$

(18)

The boundaries of each region and sub-region have been selected according to the state space partition technique of Part 4.1. For example the limit of the $R_{1,5}$ sub-region guaranties that even at the boundary point of 0.08, no switching will occur during the prediction horizon ($N = 5$) even in the worst case ($Q_1$ and $Q_2$ at the maximum value).

The possibilities of fault occurrence have also been considered, as the three tanks benchmark is subject to three faults: the valve $V_1$ may be blocked up open or closed, and the valve $V_2$ may open. Therefore, for example, the calculation of the lower bound of $R_{2,3}$ takes into consideration that the events ‘the valve $V_1$ may be blocked up open’ and ‘the valve $V_L$ may be opened’ can occur, even in this case this bound value guaranties that no switching will happen.

All this has been applied in simulation to reach the level specification previously given. The results are presented on Figure 4 for the three tanks levels and on Figure 5 for the control signals.

*Fig. 4. Water levels in the three tanks*

*Fig. 5. Controlled variables*

The level of the third tank oscillates around 0.1 as $h_3 = 0.1$ does not correspond to an equilibrium point. Consequently, the system opens and closes the two valves $V_1$ and $V_2$ to maintain the level in the third tank around the desired level of 0.1m.

As a comparison purpose between the multi-MLD models technique and the classical global MLD model strategy, the same previous level specification has been considered with a global MLD model of the benchmark, and the two prediction horizons equal to 2. Table 1 illustrates the total time required to reach the specification, the total number of QP’s solved and the maximum time to find the optimized solution. It
can be seen that the difference between the two techniques is quite large, the multi-MLD models technique allowing real time implementation and avoiding exponential explosion of the algorithm. All data given above were obtained using the MIQP Matlab code (see Bemporad and Mignone, 2000a) for solving a mixed integer quadratic programming, on a 1.8 MHz Pc with 256 kram.

Table 1 Comparison of performances obtained with the multi-MLD model and the classical MLD model.

<table>
<thead>
<tr>
<th>Approach</th>
<th>Total time</th>
<th>No of QP’s solved</th>
<th>Max. time</th>
</tr>
</thead>
<tbody>
<tr>
<td>Classical MLD</td>
<td>1323 s</td>
<td>11 912</td>
<td>236.7 s</td>
</tr>
<tr>
<td>Multi-MLD</td>
<td>68 s</td>
<td>794</td>
<td>9.9 s</td>
</tr>
</tbody>
</table>

Figures 6 and 7 respectively present the three tanks levels and the control signals with the multi-MLD models technique, where the desired level in the third tank is changing. It can be seen that the predictive control strategy enables to anticipate variations of the reference which is one of the predictive control advantages. It must be noticed that the variation of the third tank level from 0.1 to 0.15, due to the benchmark physical features.

![Fig. 6. Water levels in the three tanks – h₃ changes](image1)

![Fig. 7. Controlled variables – h₃ not constant](image2)

6. CONCLUSIONS

This paper presents a multi-MLD models structure which successfully improves the computational problem of the Mixed Logical Dynamical (MLD) formalism. A complete study of the system dynamics enables to divide the state space into regions where no switching could happen, so that a restricted number of variables is required, and other switching regions. Each region is then coupled to a specific simpler MLD model suitable for control, which enables to define particular weighting factors according to the priority of each region. All the calculation of state space partition and the development of multi-MLD models are made off-line. Future work may consider searching the state space partition taking into account several auxiliary logical variables at the same time, instead of one after the other. This may lead to less conservative results.

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